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Electronic Notes in Discrete Mathematics 41 (2013) 53–60

Electronic Notes in DISCRETE MATHEMATICS

www.elsevier.com/locate/endm

Home Care optimization: impact of pattern generation policies on scheduling and routing decisions

Paola Cappanera [1]

Dipartimento di Ingegneria dell'Informazione
Università degli Studi di Firenze
Firenze, Italy

Maria Grazia Scutellà [2]

Dipartimento di Informatica
Università di Pisa
Pisa, Italy

Abstract

In Home Care optimization, operators have to be assigned to patients by taking into account compatibility skill constraints, and patient visits have to be scheduled in a given planning horizon. Moreover, operator tours have to be determined. Integer Linear Programming models have been proposed which use the concept of patterns, i.e. a priori scheduling profiles, to combine the diverse decision levels. Computational results on real instances show that pattern generation policies are crucial to address scheduling and routing in large Home Care instances.

Keywords: Home Care, Integer Linear Programming, computational experiments.

[1] Email: `paola.cappanera@unifi.it`
[2] Email: `scut@di.unipi.it`

http://dx.doi.org/10.1016/j.endm.2013.05.075

1 The problem

Nowadays, the ever increasing average age of population and the increased costs for the consequently required care, compel the medical care units (hospitals and so on) to offer Home Care Services in an attempt to limit costs. Even more important, medical treatments carried out at patients home impact favorably on their quality of life. Therefore, Home Care Services are a cost-effective and flexible instrument in the social system.

Here we address a relevant optimization problem arising in Home Care; specifically, given a planning horizon W, usually a week, a set of patients with an associated *care plan*, i.e. weekly requests each of them demanding a specific level of skill to be operated, and a set of operators also characterized by a specific skill, the problem asks to schedule the patient requests during the planning horizon, to assign the operators to the patients by taking into account the compatibility between request and operator skills, and to determine the tour each operator has to perform in every day of the planning horizon.

More formally, the Home Care Problem (HCP) under investigation is defined on a complete directed network $G = (N, A)$, having n nodes, where each node j corresponds to a patient. There is an extra node (node 0), which is used to denote the basis of the operators. A set K of $\overline{k}$ levels of skill is assumed for both patients and operators, where skill $\overline{k}$ corresponds to the highest ability and skill 1 to the lowest one. A care plan is associated with each patient j that specifies the number of visits required by j in the planning horizon W relatively to each skill level $k \in K$. Specifically, r_{jk}, with $k \in K$, gives the number of visits of skill k required by j in W. A set O of (skilled) operators is available in the planning horizon. In addition, a subset $O_d \subseteq O$ of the operators is available on day d, for each $d \in W$. A hierarchical structure of the skill levels is assumed for the operators, so that an operator with skill k can work all the requests characterized by a skill up to k.

In HCP the scheduling of the patient requests in W, the operator assignment and the routing decisions are offered through a new modelling device, called *pattern*. We assume in fact that the patient requests are operated according to a set P of a priori given patterns. In particular, for each pattern $p \in P$, we define $p(d) = 0$ if no service is offered at day d, while it is $p(d) = k$ if a visit of skill k is operated according to pattern p on day d.

Given the input data above, HCP thus consists in assigning one pattern from P to each patient j, so scheduling the requests of j during the planning horizon (*care plan scheduling*), in assigning operators to each patient j, for each day where a request of j has been scheduled (*operator assignment*), and in determining the tour of each operator for each scheduled day (*routing decisions*). In addressing these three groups of decisions, the skill constraints (i.e. the compatibility between the skills associated with the patient requests and the skills of the operators) have to be taken into account. Other relevant Quality of Service requisites are considered.

An objective function typically used to guide the Home Care decisions is the balancing of the workload among the operators. Hence we maximize the minimum operator utilization factor, where the *operator utilization factor* is the total workload of the operator in W over his/her maximum possible workload.

In the state-of-the-art literature Home Care problems are usually solved in cascade: first the operators are assigned to the patients on a geographical basis; second, the schedule of each operator is determined, usually operator-wise. See for example [4]. Some Vehicle Routing Problem (VRP)-like formulations exist in the literature, which however generally deal with a daily planning horizon. To the best of our knowledge there are only three exceptions ([1,5,6]). However, no exact approach is proposed there to solve the overall problem, but two-stage solution approaches are presented. On the other hand, here HCP is solved by jointly addressing assignment, scheduling and routing decisions over W, by incorporating the skill hierarchy structure introduced in ([2]) where, however, only daily routing decisions have been addressed.

The HCP solution approach is based on new Integer Linear Programming (ILP) formulations, which have been proposed in [3]. As previously indicated, the innovative modelling device proposed to combine the various levels of decisions is the use of a priori given *patterns*. Three policies to generate patterns have been designed in [3] and will be discussed in this paper, by emphasizing their impact on the efficiency of the optimization approach and on the quality of the returned solutions.

2 Pattern generation policies

In order to formulate HCP, the main decision variables ([3]) are:

$$z_{jp} = \begin{cases} 1 \text{ if patient } j \text{ is assigned to pattern } p \\ 0 \text{ otherwise} \end{cases} \qquad j \in N \ (j \neq 0),\, p \in P$$

$$x_{ij}^{td} = \begin{cases} 1 \text{ if operator } t \text{ uses } (i,j) \text{ on day } d \\ 0 \text{ otherwise} \end{cases} \qquad (i,j) \in A,\, d \in W,\, t \in O_d$$

The proposed formulations to HCP include the following constraints:

$$\sum_{p \in P} z_{jp} = 1 \qquad \forall j \in N \setminus \{0\} \tag{1}$$

$$\sum_{i \in N} \sum_{t} x_{ij}^{td} \leq \sum_{p: p(d) \geq 1} z_{jp} \qquad \forall j \in N \setminus \{0\}, \forall d \in W \tag{2}$$

$$\sum_{i \in N} \sum_{t: s_t \geq k} x_{ij}^{td} \geq \sum_{p: p(d) = k} z_{jp} \qquad \forall j \in N \setminus \{0\}, \forall d \in W, \forall k \in K \tag{3}$$

$$\sum_{i \in N} x_{ij}^{td} = \sum_{i \in N} x_{ji}^{td} \qquad \forall j \in N \setminus \{0\}, \forall d \in W, \forall t \in O_d \tag{4}$$

Constraints (1) assure that each patient is assigned exactly to a pattern. Constraints (2) state that at most one operator per day can visit patient j, if a visit has been scheduled on that day for node j. By denoting with s_t the skill level of operator t, constraints (3) guarantee that, on day d, exactly one operator, of adequate skill, must visit patient j if a service has been scheduled for j on day d. This is true for each skill level k. In particular, the least skilled operators can perform only visits of skill 1 (case $k = 1$), whereas the most skilled operators can perform all types of visits (case $k = \bar{k}$). (4) are the classical flow conservation constraints on the routing variables.

(1)-(4) reveal that the pattern device has an impact on both scheduling and routing decisions, and therefore it is crucial for the efficiency of the overall optimization process, and also in determining the quality of the returned solutions. To investigate this impact three pattern generation policies have been analysed. *Heur* is a greedy heuristic procedure based on the frequency of the request types: it firstly orders the patient requests according to their numbers of requirements for increasing levels of skill and then, by scanning the ordered list, generates patterns that can accomplish with the frequency of such requests. *ImplSol* is based on the extraction of pattern information from the solution actually implemented at the Home Care provider. The third policy, called *FlowBased* or simply *FB*, is a multicommodity flow based approach defined on an auxiliary layered network $G_W = (N_W, A_W)$, with $|N_W| = n_w$, having one layer L_d for each considered day d in W, plus a source node (say 1) and a destination node (say n_w). Each layer is composed of $\bar{k} + 1$ nodes: node 0, which indicates that no visit is scheduled in the day corresponding to the layer, and a node k, for each $k \in K$, which represents the scheduling of a visit of skill k. In G_W there exists a directed arc from the source node to the nodes in first layer, from each node in the last layer to the destination node, and from each node in layer L_d to each node in the next layer, for each $d \in W$.

Any directed source-destination path in G_W corresponds to a potential pattern. Therefore, we introduce a binary commodity for each patient j, having node 1 as the origin and node n_w as its destination, and state the following multicommodity flow problem on G_W as a tool to generate patterns:

$$\min \quad \sum_{(h,i)\in A_W} q_{hi}$$

$$\sum_{(h,i)\in A_W} f^j_{hi} - \sum_{(i,h)\in A_W} f^j_{ih} = \begin{cases} -1, \text{ if } i = 1, \\ 1, \quad \text{if } i = n_w, \\ 0, \quad \text{otherwise} \end{cases} \qquad \forall i \in N_W, \forall j \in N \setminus \{0\} \tag{5}$$

$$\sum_{d\in W} \sum_{(h,k):k\in L_d} f^j_{hk} = r_{jk} \qquad \forall j \in N \setminus \{0\}, \forall k \in K \tag{6}$$

$$\sum_{j\in N\setminus\{0\}} t_j \sum_{(h,k):k\in L_d} f^j_{hk} \leq \sum_{t\in O_d:s_t\geq k} D_t \qquad \forall d \in W, \forall k \in K \tag{7}$$

$$\sum_{j\in N\setminus\{0\}} f^j_{hi} \leq n q_{hi} \qquad \forall (h,i) \in A_W \tag{8}$$

$$f^j_{hi} \in \{0,1\} \qquad \forall (h,i) \in A_W, \forall j \in N \setminus \{0\}$$

$$q_{hi} \in \{0,1\} \qquad \forall (h,i) \in A_W$$

For each patient j, the flow variables $\{f^j_{hi}\}$ model a directed path in G_W from node 1 to node n_w (constraints (5)). These variables model a feasible pattern for j, that is a pattern which is compatible with the care plan of j, thanks to constraints (6). Constraints (7) take into account, skill by skill, the operators availability in each day of the planning horizon. In fact, denoting by t_j the service time at patient j, (7) impose that the total service time of scheduled visits of skill k per day does not exceed the daily availability of the operators of skill at least k (here D_t denotes the workday length of operator t). Finally, constraints (8) link together the flow variables f^j_{hi} with the design variables q_{hi}. Such auxiliary variables $\{q_{hi}\}$ are introduced to discover which arcs are used to design the patterns: by minimizing the total number of used arcs, the model thus tends to minimize, in an implicit way, the number of generated patterns.

In the experimental campaign, policy *FB* has been used in combination with a parameter that reduces the operators availability. In fact, since the flow based model neglects the traveling times, it may occur that the patterns thus provided generate an infeasible solution when the routing issue is also considered. A reduction of the operators availability is then used as a means to prevent some undesirable infeasibilities. Specifically, three values of the aforesaid parameter are considered: 0.5 which halves the operators availability, 0.75 which reduces the availability by 25%, and 1 which maintains the real availabilities. The corresponding policies will be denoted by *FB-50*, *FB-75* and *FB-100*, respectively.

3 Computational results

We generated a set of Home Care instances starting from real data provided by one of the largest Italian public medical care unit. The considered provider operates in the north of Italy and its services cover a region that is organized in divisions, in turn organized in districts. The instances we used consider the largest district of the Merate area and comprise 10 municipalities. Two skills are considered for operators and patient requests: *ordinary*, corresponding to the lowest ability or skill 1, and *palliative*, corresponding to the highest ability or skill 2. In regards to the patients, we had access to the care profile of 4123 patients in the time period [2004 - 2008] and we selected two weeks, i.e. a week in January 2006 (hereafter denoted by *January 2006*) with 129 patients and a week in April 2007 (hereafter denoted by *April 2007*) with 163 patients. Patient demands had been computed starting from historical series. The district under consideration is characterized by 11 operators, 8 of which of skill 1 and 3 of skill 2. In all the generated instances, the traveling times have been computed via Google Maps for the inter-municipalities distances, while they have been set equal to 3 minutes for the intra-municipalities distances, consistently with the provider indications. Furthermore, according to the medical care unit indications the service time has been fixed to 30 minutes.

In regards to the patterns, which are a peculiarity of our approach, we used the three generation policies described in Section 2, by considering a weekly time horizon. The three policies may produce a number of patterns very different the one from each other; these values are reported in Table 1.

Summarizing, the experimental campaign analyzes the impact of the pattern generation policies (5 choices) in the selected weeks, both in terms of efficiency of the optimization process and quality of the solutions provided. The experiments related to the five policies have been performed on a AMD Opteron(tm) Dual Core Processor 246 (CPU MHz 1991.060). The solver is CPLEX 12.4 with a time limit of 12 hours and a memory limit for the branch and bound tree of 1 GB. In the following the computational times are ex-

Table 1
Number of patterns used

Week	Heur	ImplSol	FB-50	FB-75	FB-100
January 2006	20	29	13	11	11
April 2007	27	33	17	14	14

Table 2
LP results

		Heur	ImplSol	FB-50	FB-75	FB-100
January 2006	LPTime	9044.38	17805.47	1337.04	219.17	266.21
	LPValue	0.3552	0.3552	0.3552	0.2922	0.2922
April 2007	LPTime	17450.49	17689.80	1942.01	1702.29	1785.68
	LPValue	0.5393	0.5393	0.5393	0.5393	inf

pressed in seconds of CPU time.

Partial (and preliminary) computational results are reported below. Precisely, Table 2 reports the performance of the models in terms of time required to solve the Linear Programming relaxation (LPTime) and in terms of LP objective function (LPValue), with string "inf" denoting infeasibility. It is worth observing that the LPTime required for policies *Heur* and *ImplSol*, i.e. the policy implemented by the provider, can be quite high. On the contrary the LPTime is much more shorter for the flow based policies which are characterized by a much smaller number of generated patterns with respect to the other policies (see Table 1). Determining a good and limited set of patterns seems thus to be crucial. Particular attention should be given to the LPValue: observe that the LPValue is the same (when available) for policies *Heur*, *ImplSol* and *FB-50*, thus suggesting that the selected patterns are sufficient to obtain the same solution quality in terms of (relaxed) minimum operator utilization factor (but, as observed, with very different computational times).

In Table 3 the results obtained when the integer problem is solved are reported for all the pattern selection policies in terms of percentage relative gap computed with respect to the best upper bound obtained in the branch and bound tree (string "n.a." is used to point out that no integer solution is found). The stopping criterion that determines the algorithm termination is given in columns Stop, where T is used to indicate that the time limit has been exceeded while M is used to indicate an out of memory condition.

It is possible to observe that the *FB-50* policy allows one to determine better solutions, in terms of percentage gaps, than the ones computed by

Table 3
IP results

	Heur		ImplSol		FB-50		FB-75		FB-100	
	%Gap	Stop	%Gap	Stop	%Gap	Stop	%Gap	Stop	%Gap	Stop
January 2006	1.02	T	0.66	T	0.07	T	n.a.	T	n.a.	T
April 2007	0.48	T	0.41	T	0.17	M	0.17	T	n.a.	inf

using policies *Heur* and *ImplSol* (recall, in fact, that the LPValue is the same, see Table 2), although the problem solution required large computational time and memory resources. However a large but affordable consumption of such resources does not seem to be an issue when the focus is a difficult planning problem that has to be solved once a week, or for a still longer time horizon.

More complete computational results, which comprise the analysis of an alternative objective function, and the assessment of the quality of the returned solutions in terms of operator utilization factor and travelled time, will be discussed.

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